CE 418.3 - Design in Reinforced Concrete

MIDTERM EXAMINATION
November 7, 2002

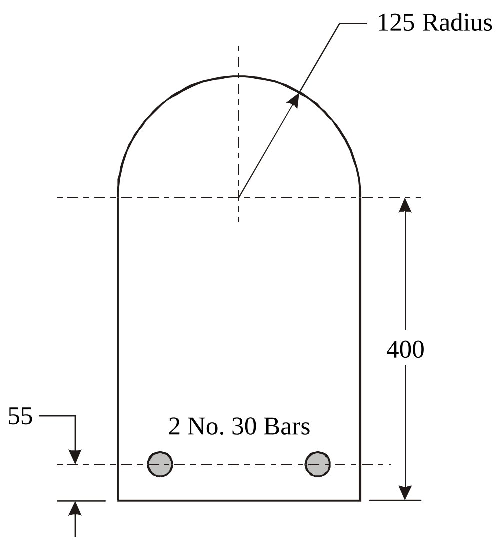
Time Allowed: 2 Hours | Professor: B. Sparling

Notes:
- Closed book examination
- CPCA Concrete Design Handbook may be used
- Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (i.e. formulas)
- Show **all** your work, including all formulas and calculations

MARKS

26 **QUESTION 1:** The reinforced concrete beam shown below features a semi-circular top poured integrally with a rectangular base. It is constructed using concrete with $f_c' = 25$ MPa and Grade 400 reinforcing steel. If the beam is subjected to positive bending moments, calculate the ultimate positive bending moment resistance M_r of the beam in accordance with CSA A23.3-94 Clause 10.1.7 (i.e. equivalent rectangular concrete stress distribution).

Hints: The compressive stress block in the concrete extends below the semi-circular top into the rectangular base. Assume that the steel yields and prove that assumption; however, do not repeat your calculations if that assumption proves incorrect.



Geometric properties of a semi -circular area:

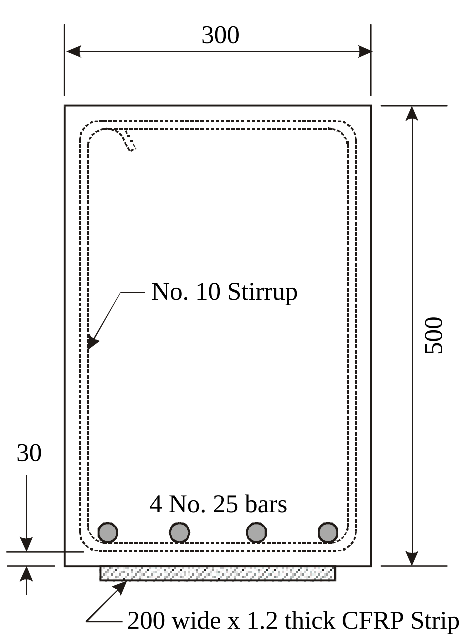
$$\text{Area} = \frac{\pi r^2}{2}$$

- Location of centroid relative to base of semi-circle:

$$c_1 = \frac{4r}{3\pi}$$

where: r = Radius of the semi-circle.

26 **QUESTION 2:** A reinforced concrete bridge girder was constructed using concrete with $f_c' = 30$ MPa and **Grade 350** reinforcing steel. To provide additional strength, a Carbon Fibre Reinforced Polymer (CFRP) strip was bonded to the bottom face of the girder when it was in a completely unloaded condition (i.e. there was no flexural strain on the bottom face of the girder when the CFRP was applied). The bond between the girder and the CFRP strip is adequate to ensure full composite behaviour.



CFRP exhibits **linearly elastic behaviour** right up to the point of failure. Data from the manufacturer specifies that the CFRP has an elastic modulus of $E_{CFRP} = 165{,}000$ MPa and an ultimate tensile strength of 2,800 MPa. The material resistance factor for CFRP may be taken as $\phi_{CFRP} = 0.7$.

Estimate the ultimate (positive) moment capacity of the CFRP-reinforced beam in accordance with the principles outlined in CSA-A23.3-94.

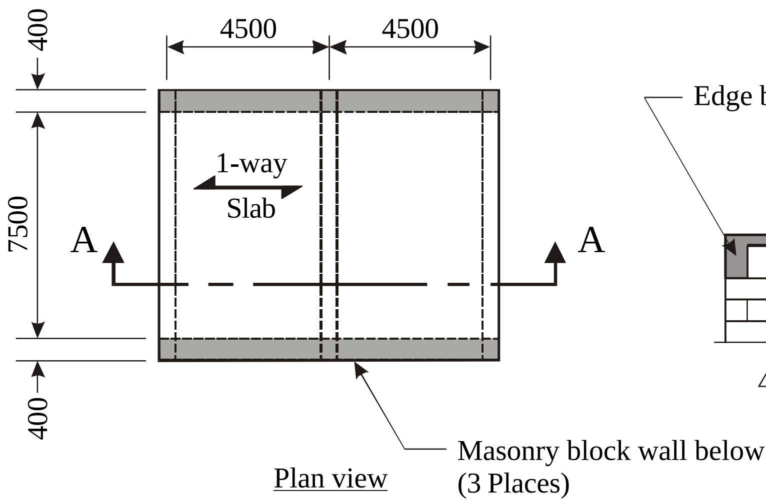
Assume that the reinforcing steel yields and prove that assumption; however, do not redo the calculations if that assumption proves incorrect.

30 **QUESTION 3 :** A continuous 1-way slab is cast integrally with three support beams as shown below. The beams are simply supported by masonry walls at both ends. The concrete has a design strength of $f_c' = 30$ MPa and the reinforcement is Grade 400. Clear concrete covers of 30 mm are to be used for the beams, along with No. 10 stirrups. The slab supports a live area load of 3.6 kPa.

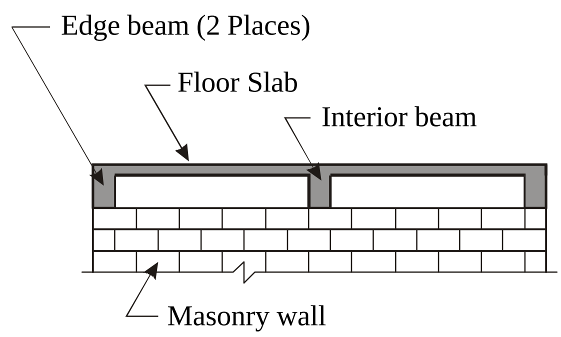
Using the design aids in the CPCA Handbook (i.e. Table 2.1 on page 2-13), select the main reinforcement for the singly reinforced **edge beams** (both edge beams will be the same).

Also, demonstrate that Table 2.1 is applicable for this case.

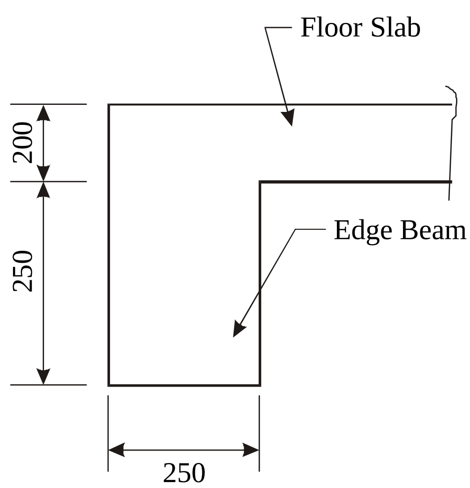
Slab reaction forces for both the dead load and critical live load pattern for the edge beam are provided below. Here, the slab span length L_s is defined as the centre-to-centre distance between support beams.



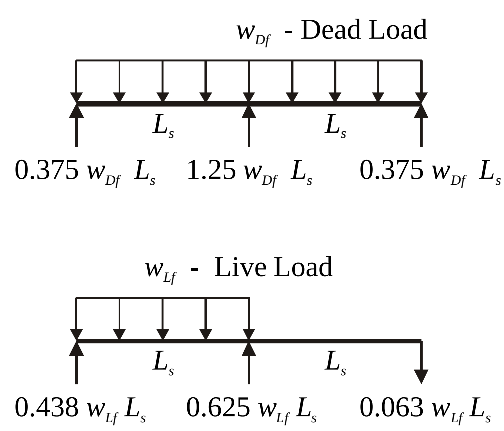
Plan view



Section A-A



Detail of edge beam



Critical slab reactions from unit design strip

QUESTION 4 : Provide **brief** answers to the following questions. Answers in point form are acceptable. Sketches should be used to supplement written responses if appropriate.

4 **a)** Compare the approaches used to define the ultimate factored resistance in reinforced concrete members in the current Canadian CSA-A23.3-94 Limit States Design (LSD) method and the American ACI-318-95 Ultimate Strength Design (USD) method.

6 **b)** List the three main benefits associated with the use of compression steel reinforcement in the design of reinforced concrete beams.

3 **c)** In terms of structural behaviour, what is implied by the assumption of **strain compatibility** within reinforced concrete beams?

5 **d)** In general terms (no equations) define what is meant by a **balanced** failure condition in a reinforced concrete beam. Also, describe the structural design significance associated with this term.

<u>Supplemental Material:</u>

- **Material Properties:** $\phi_c = 0.6$ $\phi_s = 0.85$ $\alpha_D = 1.25$ $\alpha_L = 1.5$

$$f'_{ct} = \frac{t}{\alpha + \beta t}\, f'_c \qquad \frac{f_c}{f'_c} = 2\left(\frac{\varepsilon_c}{\varepsilon'_c}\right) - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2 \qquad f_{ct} = \frac{2P}{\pi d L} \approx 0.53\sqrt{f'_c}$$

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\gamma_c/2300\right)^{1.5} \qquad E_s = 200{,}000 \text{ MPa} \qquad \varepsilon_{cu} = 0.0035$$

$$f_r = 0.6\,\lambda\sqrt{f'_c} \qquad \gamma_c = 2400 \text{ kg/m}^3$$

- **Flexural Analysis:** $\Sigma F_x = 0$ $\Sigma M = 0 \rightarrow M = T\,(j\,d) = C_c\,(j\,d)$

$$C_c = \int_0^c f_c \,\mathrm{d}A \qquad \bar{y}\, C_c = \int_0^c y\, f_c \,\mathrm{d}A \qquad C_c = \left(\phi_c\ \alpha_1\ f'_c\right)(\text{Area}) \qquad T = \phi_s\ A_s\ f_s$$

$$\alpha_1 = 0.85 - 0.0015\ f'_c \geq 0.67 \qquad \beta_1 = 0.97 - 0.0025\ f'_c \geq 0.67 \qquad = \beta_1\, c$$

$$a = \frac{\phi_s\ A_s\ f_s}{\phi_c\ \alpha_1\ f'_c\, b} \qquad \varepsilon_s = \varepsilon_{cu}\left(\frac{d-c}{c}\right) \qquad \frac{c}{d} \leq \frac{700}{700 + f_y} \qquad \frac{d'}{c} \leq 1 - \frac{f_y}{700}$$

$$\left(A_s\right)_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1\, b\, d}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad A_{s1} = A'_s\left(\frac{f'_s}{f_s} - \frac{\phi_c\ \alpha_1\ f'_c}{\phi_s\ f_s}\right) \qquad A_{s2} = A_s - A_{s1}$$

$$M_{r1} = \phi_s\ A_{s1}\ f_{s1}\left(d - d'\right) \qquad M_{r2} = \phi_s\ A_{s2}\ f_{s2}\left(d - \frac{a}{2}\right) \qquad \varepsilon'_s = \varepsilon_{cu}\left(\frac{c - d'}{c}\right)$$

- **Flexural Design:** $A_{s_{\min}} = \dfrac{0.2\sqrt{f'_c}}{f_y}\ b_t\ h$ $\rho = \dfrac{A_s}{b\,d}$ $K_r = \dfrac{M_r \times 10^6}{b\,d^2}$

$$\rho_{\text{bal}} = \frac{\phi_c\ \alpha_1\ f'_c\,\beta_1}{\phi_s\ f_y}\left(\frac{700}{700 + f_y}\right) \qquad K_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right) \qquad M_r \geq M_f$$

$$M_r = \phi_s\ \rho\ f_y\left(1 - \frac{\phi_s\ \rho\ f_y}{2\,\phi_c\ \alpha_1\ f'_c}\right) b\, d^2 \qquad \rho = \frac{\phi_c\ \alpha_1\ f'_c \pm \sqrt{\left(\phi_c\ \alpha_1\ f'\right)^2 - 2\,K_r\,\phi_c\ \alpha_1\ f'}}{\phi_s\ f_y}$$

- **One-Way Floor Systems:** $A_{s_{\min}} = 0.002\ A_g$ $A_{sh} = \dfrac{\left(\phi_c\ \alpha_1\ f'_c\right)\left(h_F\ b\right)}{\phi_s\ f_y}$